ABSTRACT

Refrigeration can be costly in terms of equipment and energy, and if not done correctly will fail to achieve its objectives and lower the quality and safety of the product. To ensure that refrigeration is effective, we need to be able to calculate processing times, product temperatures, heat loads and water diffusion into and out of the product.

The calculation of food cooling and freezing may be complicated by phase change. The product undergoing cooling often has very complex shape and composition. Heat transfer may be coupled with moisture transfer and the equations governing the two processes should be solved simultaneously. The heat transfer coefficient is often difficult to determine for the infinite variety of real-life situations such as packaged products, cryogenic cooling, highly turbulent flow, swirling and non-parallel flow, etc. and within the same chiller the heat transfer coefficient will vary greatly from place to place. The heat transfer coefficient may also vary along the surface of a product due to the complex turbulent flow pattern and the development of the flow boundary layer, a problem still largely unsolved even by the most advanced computational fluid dynamics packages. Food products often have inconsistent compositions, shapes and sizes, which lead to variable thermal behaviour and quality after processing, so that it is difficult to quantify accurately the effects of different processing practices.

This paper will review methods for modelling foods with complex shapes, from highly simplified models to detailed numerical models including CFD. Typical results on the accuracy of various methods will be reported.

Keywords:

Cooling time, freezing time, product heat load, modelling

1. INTRODUCTION

From the refrigeration engineer’s point of view, the most important parameters determining the design of cooling and freezing processes and equipment are the processing time and the heat load. The former determines the size of the processing equipment (chiller, freezer) since holding capacity is the product of throughput and residence time, while the latter determines the capacity of the refrigerating equipment (compressors, condensers) and the energy consumption.

Twenty years ago, the refrigeration engineer would estimate processing time and product heat load from experience or, if he was really "nerdy", by using some simple empirical equation or chart. There is a lot of uncertainty in using such methods but as products and processing methods tended to change only slowly, design engineers had time to experiment and learn from their experience.

In the last twenty years, things have changed greatly. New markets and products are being developed more and more quickly, while computers have also progressed at an increasing rate, doubling in speed and capacity every three years. Commercial calculation packages such as finite element methods and computational fluid dynamics have appeared in the market, giving the designer new tools that are very
powerful but also at times dangerous to use. This paper will attempt to give a review of the range of computational methods that are now available.

2. THEORY

During the chilling and freezing of solids, heat must be conducted through layers of material before reaching the surface, then cross over to the surrounding fluid either directly or through layers of packaging. Processing time and heat load depend on the properties of the product (thermal conductivity, density and specific heat), the surface heat transfer coefficient and the thermal resistance of any packaging, and also on the size and shape of the product. The latter is often a major obstacle to the accurate prediction of processing parameters, due to the complex shape of many products of interest.

The processing time is usually defined as the time for the thermal centre of the product to reach a certain temperature. In a complex geometry, it depends on what happens in the thickest part of the object, for example the rump of a carcass. The heat load varies greatly during processing, being highest at the start, and this peak heat load, which may determine refrigeration requirements, often depends on what happens in the thinnest parts (biggest surface area per unit volume). Only in objects of simple, regular shapes (spheres, long cylinders, large slabs) will both be determined by the same dimension.

When the internal resistance to heat transfer is negligible compared to the surface resistance, the whole object will be at uniform temperature and the calculation of processing time and heat load becomes trivial. In the more general case the Biot number, $Bi$, measures the ratio of internal to external resistance:

$$Bi = hR / k$$

where $h$ is the surface heat transfer coefficient (including the effect of any packaging, radiation and evaporative cooling), $R$ the smallest half-dimension and $k$ the thermal conductivity of the product. The Biot number is of fundamental importance to any study of heat transfer to solid objects. Internal resistance can be neglected if $Bi << 1$ (uniform internal temperature). Another limiting case is when $Bi$ tends to infinity, for which analytical solutions have been found for many cases. Many simplified methods use interpolation to predict cooling times and heat loads for intermediate cases.

It is vital for practicing engineer to be aware of the Biot number of their thermal processes. If it is much larger than 1 (i.e. if internal resistance is controlling heat transfer), then there is little use trying to improve heat transfer say by reducing packaging or increasing air velocity, and more advantage will be gained by trying to reduce the dimensions of the product (if possible); the inverse applies when $Bi$ is much smaller than 1.

3. SIMPLIFIED CALCULATION METHODS

Cooling time

When a solid object is cooled from a uniform initial temperature $T_i$ in an medium at constant temperature $T_a$, the temperature $T$ at any location in the solid as well as the mean temperature obey the following equation, which is a sum of exponential decay terms (Carslaw and Jaeger [1]):

$$\frac{(T - T_a)}{(T_i - T_a)} = c_1 \exp(-b_1t) + c_2 \exp(-b_2t) + ...$$

where the coefficients $b_i$ and $c_i$ depend on the Biot number and the location in the product. The series solution is difficult to compute but graphs of the solutions are provided in many heat transfer textbooks for some simple shapes (e.g. Holman [2] p.148). However, the situation is greatly simplified if the temperature fall is more than about 30% of the maximum that can be achieved (i.e. $(T - T_a)/(T_i - T_a) \leq 0.7$). In that case, all the terms in the series solution become negligible except for the first, which has
the smallest decay rate. Thus, after the initial period the residual temperature difference and heat load decay exponentially. The centre temperature $T_c$ and mean temperature $T_m$ are described during the exponential decay phase by

\[
\frac{(T_c - T_a)}{(T_i - T_a)} = j_c \exp\left(-\frac{2.303 t}{f}\right)
\]

(3)

\[
\frac{(T_m - T_a)}{(T_i - T_a)} = j_m \exp\left(-\frac{2.303 t}{f}\right)
\]

(4)

from which cooling time can be calculated. $f$ is the time for the residual temperature difference or heat load to decrease by a factor of 1/10 during the decay period, while $j$ is called the "lag factor" (it measures how much the actual decay curve lags behind a true exponential decay curve). While the $j$-factor depends on the position in the product, $f$ remains the same for all positions.


For complex or irregular shapes, there has been two main lines of approach to the prediction of cooling time. The first is to approximate the object by an idealised (simple) shape with an equivalent thickness or diameter. Thus, a lamb leg or beef rump can be approximated by an equivalent infinite cylinder [6,7] or a beef carcass can be described as a collection of slabs and cylinders [8]. As long as the real shape does not deviate too far from the ideal, the equivalent dimension can be calculated from, say, the volume/area ratio.

The second approach attempts to derive general expressions for the $j$ and $f$ factors of complex shapes from their various dimensions. The most advanced of these methods is that of Lin et al. [9-11]. The formulas are long and involved but simple to apply on a spreadsheet.

**Heat load during cooling**

For simple shapes, analytical and graphical solutions can be found in most heat transfer textbooks. The $f$-$j$ method should normally be avoided since it does not apply in the initial cooling phase, which is when heat load is highest. However, if the average heat load $Q_{av}$ between times 0 and time $t$ is needed, it can be calculated from

\[
Q_{av} = mc \frac{(T_i - T_m)}{t}
\]

(5)

where $T_m$ can be found by the $f$-$j$ method, as long as the exponential period has been reached. For example, the average heat load $Q_{0.07}$ between time 0 and time $f \log_{10} (j_m/0.7)$, at which the residual mean temperature difference has fallen to 70% of its initial value, is

\[
Q_{0.07} = 0.3 \frac{mc (T_i - T_a)}{[f \log_{10} (j_m/0.7)]}
\]

(6)

**Freezing time**

The freezing of a food can be divided into three distinct periods:

1. A precooling period, where cooling takes place without phase change.
2. A phase change period, which starts when the surface reaches the freezing point $T_f$ and ice forms at the surface. The freezing front gradually advances into the product, until it reaches the thermal centre and the whole food can be considered frozen.
3. A postcooling period, during which the whole food continues to cool below $T_f$.

Plank [12,13] presented an analytical solution for the phase change period only. For many years, Plank's equation was the only one available and it is still taught in many textbooks. However, it can lead to large errors (typically 30%). There is no analytical solution for food freezing that includes all three periods. Furthermore food freezing is more complicated than the freezing of pure water, because the water in the food does not freeze instantaneously but does so over a range of temperature. To
predict freezing time for a simple shape, many empirical equations have been presented in the last
twenty five years. A fairly simple equation which is nevertheless quite accurate (±10%) is that by the
author (Pham [14]):

\[
t_f = \frac{R}{E_{\text{Freeze}} h} \left( \frac{\Delta H_1}{\Delta T_1} + \frac{\Delta H_2}{\Delta T_2} \right) \left( 1 + \frac{\text{Bi}}{2} \right)
\]

(7)

where \( \text{Bi} = hR/k_f \), \( E_{\text{Freeze}} \) is the equivalent heat transfer dimensionality (EHTD) shape factor for
freezing (1 for slab, 2 for infinite cylinders, 3 for spheres), \( \Delta H_1 \) and \( \Delta T_1 \) are the volumetric
enthalpy change and temperature difference respectively for the precooling period, and \( \Delta H_2 \) and \( \Delta T_2 \) those for
the combined freezing-postcooling period.

As in cooling, the freezing of an irregular shape can be approximated by that of an equivalent regular
shape; for example, an ellipsoid can be approximated by an equivalent sphere. The technique has also
been frequently used on specific products such as lamb legs.

Another approach for complex shapes is to first calculate the freezing time for a slab then divide it by
the shape factor \( E_{\text{Freeze}} \), which is available for many shapes (Hossain et al. [15, 16], Cleland et al.
[17,18], McNabb et al. [19,20]).

### Freezing heat load

A simple method for calculating freezing heat loads was proposed by Lovatt et al. [21,22] (Figure1). It
is based on how the freezing front moves towards the thermal centre and involves an empirical shape
factor \( N \), which may be different from the shape factor \( E \) for freezing time (because heat load depends
what happens in the thinnest parts of the product while freezing time depends on what happens in the
thickest parts).

\[
\frac{dx_i}{dt} = \frac{T_c - T_i}{L \epsilon^2 \left[ \frac{1}{h \lambda^n} \frac{(x_i^{1-n} - x_i^{1-n})}{k_i(1-n)} \right]}
\]

Figure 1. Lovatt et al's method for freezing heat load calculation.

\[
\frac{dV}{dx_i} = N \left( \frac{x_i^{n-1}}{x} \right) \frac{V}{X}
\]

\[
\phi = \frac{dH}{dt} = \frac{dx_i}{dt} \frac{dV}{dx_i}
\]

### Calculating Product Heat Load in an Industrial Cooler or Freezer

The methods described in this chapter enable the calculation of heat load from a single item of product.
However, in industry normally many items will be processed at one time, and it is important to take
into account the operational characteristics of the process. For a strict batch operation, when the cooler
is filled with product before refrigeration is turned on, the refrigeration must be able to cope with the
peak heat load under the starting condition. For a continuous operation, the heat load is constant and
can be calculated from the mean temperatures of the entering and leaving products and the throughput.
Many operations in the food industry operate in an intermediate mode; for example, a chiller or freezer
may be loaded/unloaded continuously during daytime and then continue to cool batchwise during the
night. At any time \( t \), the product in the cooler will have residence times ranging from 0 to \( t_{\text{max}} \) and
hence varying heat loads. In such situations, the total heat load at time \( t \) is made up of the heat load
from product loaded at all times between \( t - t_{\text{max}} \) and \( t \). The time from \( t - t_{\text{max}} \) to \( t \) can be divided into
a number of intervals, the heat load from product loaded in each interval calculated and added together.
4. NUMERICAL (DISCRETIZATION) METHODS

In numerical discretization methods (finite differences, finite elements, finite volume), the product is divided into small control volumes or elements. The heat conduction equation is written for each of these control volumes/elements, giving a set of hundreds, thousands or sometimes millions of equations. These equations are then solved together to calculate the change with time of the temperature in each location.

Finite differences (FD) was the earliest such method. The product is represented by a regular grid and the equations for heat flow between the "nodes" (grid points) are written out and solved by computers. Nowadays, the solution of a finite difference problem on a computer is very fast, of the order of a few second or less. The finite difference equations can be set up and solved by any mathematically competent engineering graduate (sad to say, not all engineering graduates are mathematically competent). However, this method is practical only for objects that have a reasonably regular shape, such as a rectangular box or a finite cylinder.

Finite Elements (FE) is preferred to finite differences for modelling objects with complex, irregular shapes. A grid is still used but it can be irregular, consisting of triangles, distorted rectangles or volumes of various shapes. The grid can be in two or three dimensions. Finite elements models take more time to solve than finite difference models due to the larger bandwith of the matrix equations that need to be solved. On a modern PC, a two-dimensional FE model may take several minutes to solve while a three-dimensional FE model may take an hour or more. Also, it is more difficult to set up the model, usually requiring special graphical software. However this is not due to the nature of the model but to the complex shape that such models are applied to. FD models are easier to set up but that is only because the real shape is usually approximated by a more regular shape.

The theory behind finite elements is mathematically abstract, and nowadays the method of finite volumes (FV), which combines the flexibility of finite elements with the conceptual simplicity of finite differences, has become widely popular. The object is divided into small control volumes of arbitrary shape (as in finite elements), and the equations of conservation for heat, mass and momentum are written for each control volumes. Computational effort is similar for FV and FE. FV is widely used in computational fluid dynamics (CFD) models.

5. CFD METHOD

One of the big uncertainties in calculating process time and heat load is the heat transfer coefficient at the surface of the product. Often the air is highly turbulent, the shape is complex giving rise to boundary layer variation and eddies, which make the heat transfer coefficient vary from place to place and become highly unpredictable. In other cases such as cartoned product, the presence of air gaps with irregular shapes containing natural convection cells make the effective heat transfer coefficient difficult to predict. In these cases the technologist must often carry out a large number of experiments, or make gross approximations based on experience.

Computational Fluid Dynamics offer the promise of eliminating these uncertainties, by calculating the flow pattern and heat transfer at the product surface from first principles, using the basic equations of heat, mass and momentum transfer. This has become possible owing to the huge computing power of modern computers.

For the case of laminar flow, CFD has indeed lived up to its promise. However, most industrial situations involve turbulent flow, for which CFD is still unable to entirely deliver the goods. This is because turbulent flow involves stochastic variations in the flow and temperature pattern, which must be averaged out, and during this averaging process, empirical equations and coefficients must be introduced and the equations lose their fundamentality. They must incorporate turbulence models which are still conceptually suspect and involve a large number of empirical coefficients. The empirical
coefficients are specific to each flow situation and if they are used beyond these (i.e. in almost every new situation practice), accuracy is no longer guaranteed.

To illustrate these points, Pham and Nguyen's [23] CFD simulation results for heat transfer coefficients during beef chilling is presented in Figure 2, using the standard $k$-$\varepsilon$ model and the RNG (ReNormalisation Group Theory) $k$-$\varepsilon$ model respectively. Both of these are empirical models with several empirical coefficients.

Another obstacle against the widespread use of CFD models is the large amount of developmental time and computation effort involved. It took about three months of an experienced user's time to build the finite volume of a beef side model, and simulating a 20-hour run took a week on a 300-MHz personal computer, with frequent intervention from the user to optimise relaxation factors etc. (and that's ignoring mass transfer).

Nevertheless, computer power continues to increase quickly, and one can expect that more fundamental turbulence models (such as the large eddy simulation model) will eventually become practicable for the food industry and yield reliable results.

**Pseudo-steady state CFD simulation**

In view of the large computing time required to simulate a transient problem, a good compromise is to use CFD to calculate the heat transfer coefficient for a steady state situation, which is much faster and takes minutes instead of days. Since the htc varies only slowly with time, it can be assumed to remain the same for a significant fraction of the process time. Thus, changes in temperature can be calculated using a conduction model (FD, FE or FV) for the inside of the product only, which is again much faster than a full transient CFD calculation. Figure 3 shows such a calculation for beef chilling (Pham and Nguyen [23]).

![Figure 2](image1.png)  ![Figure 3](image2.png)

**Figure 2.** Heat transfer coefficient predicted using the standard $k$-$\varepsilon$ model (dotted curves) and the RNG $k$-$\varepsilon$ model.

**Figure 3.** Leg centre and surface temperatures of beef side. ——— calculated by CFD using the RNG turbulence model, ••• measured.

6. **ACCURACY OF COOLING AND FREEZING PREDICTION METHODS**

In view of the large range of calculation methods available it would be useful to compare them against each other and against empirical data, where available. This will help the refrigeration engineer and food engineer decide which to use should the need arise.
Errors in predicting cooling time

Prediction errors are due to the following sources:

- errors inherent the calculation method (for example, by approximating the cooling curve by the f-j exponential)
- errors in the product’s thermal properties (thermal diffusivity)
- errors in the heat transfer coefficient – effect of turbulence, radiation, evaporation, wrapping etc.
- errors in predicting the effect of complex shape

The first source of errors can be eliminated by using a strict analytical or numerical method, however the other sources of errors cannot be entirely eliminated. Surprisingly little data is available on the extent of these errors in practice. However, we can expect that they will be of the same order as those incurred during freezing calculations, for which data are available, as will be seen below.

Errors in predicting freezing time

Cleland [24] did some careful analysis of the most well known freezing time prediction methods (Table 1). It can be seen that the most accurate methods, such as that of Pham presented above, can predict freezing time to about ±10%. There is also quite good agreement between simplified formulas and numerical methods such as finite differences.

<table>
<thead>
<tr>
<th>Method</th>
<th>Mean (%)</th>
<th>SD (%)</th>
<th>Range (%)</th>
<th>Correl.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Finite differences</td>
<td>0.1</td>
<td>7.6</td>
<td>-11 to 12</td>
<td>—</td>
</tr>
<tr>
<td>Cleland &amp; Earlis (1984)</td>
<td>-0.2</td>
<td>6.3</td>
<td>-10 to 10</td>
<td>0.728</td>
</tr>
<tr>
<td>Hung &amp; Thompson (1983)</td>
<td>-1.7</td>
<td>8.7</td>
<td>-9 to 21</td>
<td>0.373</td>
</tr>
<tr>
<td>Eqn A2 of Cleland et al. (1987)</td>
<td>-0.9</td>
<td>6.7</td>
<td>-13 to 9</td>
<td>0.728</td>
</tr>
<tr>
<td>de Michelis &amp; Cepolo (1983)</td>
<td>-1.7</td>
<td>11.4</td>
<td>-17 to 18</td>
<td>0.858</td>
</tr>
<tr>
<td>Pham (1984)</td>
<td>0.4</td>
<td>7.1</td>
<td>-11 to 9</td>
<td>0.787</td>
</tr>
<tr>
<td>Pham (1986a)</td>
<td>1.1</td>
<td>7.3</td>
<td>-11 to 12</td>
<td>0.772</td>
</tr>
</tbody>
</table>

Table 1. Comparison of simplified freezing time prediction methods with experiments and finite difference calculations.

It must be noted that these error values apply to experiments carried out in the laboratory under carefully monitored conditions, with well known test materials, so that the errors due to heat transfer coefficients, product shape and material properties are minimal. In industry, larger errors can be expected, although with some expertise, errors of perhaps no more than ±15% can be achieved.

Errors in predicting cooling and freezing heat load

The author and his associates at MIRINZ, Massey University (NZ) and the University of New South Wales seem to have been the only group to have gathered reasonably accurate heat load data during cooling and freezing. These data have been obtained using a flow calorimeter technique [25] (Figure 4). The hot product, for example a carton of meat or a beef side, is put in a wind tunnel. A thermopile (a sensitive differential temperature sensor) is made by having an array of thermocouples distributed over the inlet flow area and another over the outlet flow area. The thermocouple pairs are connected in series, thus multiplying the thermoelectric signal. For example, with 20 thermocouple pairs, a voltage signal of about 8 µV is obtained for every 0.01°C temperature rise in the air, which can be easily measured with a digital voltmeter accurate to 1µV. There is some errors introduced by heat loss through the tunnel walls, but as long as conditions are stable these effects can be removed by a simple baseline correction.

A typical heat load curve for beef chilling (Davey and Pham [8]) is shown in Figure 5. Davey and Pham [26] did a comparison of predicted and measured heat load (averaged over the first two hours of
chilling) for 55 beef chilling tests (Table 2). It can be seen that the predictions for both a finite differences model and a 2-D finite element models are quite acceptable, although there is a definite improvement with the FE model (5.6% average error in heat load for FE vs 12.6% for FD).

Table 1. Comparison of FD and FE model against experimental data for 55 tests.

<table>
<thead>
<tr>
<th></th>
<th>FD Model</th>
<th>FE Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Representation of beef side</td>
<td>7 simple-shaped regions</td>
<td>13 cross-sections</td>
</tr>
<tr>
<td>Average % error in heat removed during first 2 hours</td>
<td>-12.6 %</td>
<td>-5.6 %</td>
</tr>
<tr>
<td>Average % error in weight loss after 20 hours of chilling</td>
<td>-1.25%*</td>
<td>2.32 %*</td>
</tr>
<tr>
<td>Time to simulate 20 hour process on Pentium 166 Mhz Computer</td>
<td>&lt; 1 min</td>
<td>4-5 hours</td>
</tr>
</tbody>
</table>

7. CONCLUSIONS

A wide range of methods for predicting cooling/freezing times and heat loads, from simple approximate equations to complex CFD models. Although sophisticated, theory-based methods are becoming more practical with the fast progress in computer hardware and software, simplified methods can still be useful in many cases. CFD is still basically a research tool in view of the major effort involved in setting up the simulation, large computing time and continuing difficulty in dealing with complex turbulent flow situations. For the majority of practical cases, a good practical compromise is the use of finite differences or finite elements code for conduction inside the product, combined with heat transfer coefficients obtained from empirical equations or CFD simulations.

REFERENCES

[3] Pflug, I.J., Blaisdell, J.L., Kopelman, I.J. Developing temperature-time curves for objects that can be approximated by a sphere, infinite plate or infinite cylinder. ASHRAE Transactions; 1965; 71; 238-48.